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CP-odd anomalous W-boson couplings from supersymmetry

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Abstract. The supersymmetric standard model contains a new CP-violating phase in the mass matrices for charginos and neutralinos, which could induce CP-odd anomalous couplings for the WWZ and $WW\gamma$ vertices at the one-loop level. We study these couplings, paying attention to the model-parameter and q^2 dependencies. It is shown that the CP-odd form factors could have values of order $10^{-3} - 10^{-4}$, which are much larger than those predicted by the standard model. We also discuss the possibility of examining these form factors in experiments.

1 Introduction

The study of trilinear gauge-boson vertices WWZ and $WW\gamma$ is one of the main subjects for experiments at LEPII or near-future e^+e^- colliders. Their precise measurements enable to examine the standard model (SM) in which the vertices are determined uniquely. Possible discrepancies between the experimental results and the SM predictions would imply the existence of physics beyond the SM [1,2]. Various theoretical analyses therefore have been made on the vertices in the SM [3] and in its extensions, such as the two-Higgs-doublet model [4], the model with Majorana neutrinos [5], and the supersymmetric model [6], in particular for CP-conserving couplings.

In this paper we study the trilinear gauge-boson vertices focusing on CP violation within the framework of the supersymmetric standard model (SSM) [7]. This model contains new sources of CP violation as well as the standard Kobayashi-Maskawa mechanism. As a result, the W and Z bosons have CP-violating interactions with supersymmetric particles [8]. These interactions generate CP-violating couplings for WWZ and $WW\gamma$ at the one-loop level [9,10]. Since the SM does not predict CP violation for the vertices at the tree level nor the one-loop level, observation of CP-violating phenomena arising from these vertices could immediately indicate the existence of physics other than the SM [11]. The SSM gives radiative corrections also to CP-even couplings for the vertices [12], although they are at most of the same order of magnitude as the SM predictions [6].

The new sources of CP violation of the SSM can also give contributions to the electric dipole moments (EDMs) of the neutron and the electron through one-loop diagrams mediated by the charginos, neutralinos, or gluinos, together with the squarks or sleptons. For wide ranges of SSM parameters the magnitudes of the induced EDMs could be around or even larger than their present experimental upper bounds, thus providing non-trivial constraints on the SSM. We assume that CP-violating phases intrinsic in the SSM have a natural magnitude of order unity, since there is no convincing reason which suppresses them. Then, the masses of the squarks and sleptons are constrained from the EDMs to be larger than 1 TeV, while the charginos and neutralinos could have masses of order of 100 GeV [13].

The one-loop diagrams which induce the CP-violating couplings for WWZ and $WW\gamma$ in the SSM could be mediated by various supersymmetric particles. However, if the new CP-violating phases are not suppressed, the squarks and sleptons have to be heavy and thus the diagrams with these particles can be neglected. Sizable contributions to the couplings could only be generated through the diagrams mediated by the charginos and neutralinos shown in Fig. 1, on which our analyses are concentrated throughout this paper.

This paper is organized as follows. In Sect. 2 the CP-violating interactions in the SSM are briefly summarized. In Sect. 3 we obtain the CP-odd form factors for the WWZ and $WW\gamma$ vertices and make numerical analyses in detail. The possibility of detecting the CP-odd couplings is discussed in Sect. 4.

2 Model

The CP-odd couplings for the W bosons are induced by the interactions of charginos ω_i and neutralinos χ_j, the

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$$
\begin{aligned}
&\times \left(-M_W^2 A_3 + (m_{\omega j}^2 - m_{\omega i}^2) S_3\right) \\
&+ \mathrm{Im}\left[G_{Lki} G_{Lkj}^* F_{Rij} + G_{Rki} G_{Rkj}^* F_{Lij}\right] \\
&\times m_{\omega i} m_{\omega j} A_1 \\
&+ \mathrm{Im}\left[G_{Lki} G_{Rkj}^* F_{Lij} + G_{Rki} G_{Lkj}^* F_{Rij}\right] \\
&\times m_{\omega j} m_{\chi k} S_2 \\
&+ \mathrm{Im}\left[G_{Lki} G_{Rkj}^* F_{Rji} + G_{Rki} G_{Lkj}^* F_{Lij}\right] \\
&\times (-m_{\omega i} m_{\chi k} S_2)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{4(b)}^Z = &\frac{-1}{(4\pi)^2} \frac{g^2}{\cos^2\theta_\mathrm{W}} \frac{1}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{2} \\
&\times \Big\{ \mathrm{Im}\left[G_{Lik} G_{Ljk}^* F_{ji} - G_{Rik} G_{Rjk}^* F_{ji}^*\right] \\
&\times \left(-M_W^2 A_3 + (m_{\chi j}^2 - m_{\chi i}^2) S_3\right) \\
&+ \mathrm{Im}\left[G_{Lik} G_{Ljk}^* F_{ji}^* - G_{Rik} G_{Rjk}^* F_{ji}\right] \\
&\times \left(-m_{\chi_i} m_{\chi_j} A_1\right) \\
&+ \mathrm{Im}\left[G_{Lik} G_{Rjk}^* F_{ji} - G_{Rik} G_{Ljk}^* F_{ji}^*\right] m_{\chi j} m_{\omega k} S_2 \\
&+ \mathrm{Im}\left[G_{Lik} G_{Rjk}^* F_{ji}^* - G_{Rik} G_{Ljk}^* F_{ji}\right] \\
&\times m_{\chi i} m_{\omega k} S_2\Big\},
\end{aligned}
$$

$$
f_6^Z = f_{6(a)}^Z + f_{6(b)}^Z, \tag{10}
$$

$$
\begin{aligned}
f_{6(a)}^Z = &\frac{1}{(4\pi)^2} \frac{g^2}{\cos^2\theta_\mathrm{W}} \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{4} \\
&\times \Big\{ \mathrm{Im}\left[G_{Lki} G_{Lkj}^* F_{Lij} - G_{Rki} G_{Rkj}^* F_{Rij}\right] \\
&\times \left(-M_W^2 (A_3 - 2A_2) - 2q^2 A_4 + 3(m_{\omega j}^2 - m_{\omega i}^2) S_3\right) \\
&+ \mathrm{Im}\left[G_{Lki} G_{Lkj}^* F_{Rij} - G_{Rki} G_{Rkj}^* F_{Lij}\right] \\
&\times m_{\omega i} m_{\omega j} A_1 \\
&+ \mathrm{Im}\left[G_{Lki} G_{Rkj}^* F_{Lij} - G_{Rki} G_{Lkj}^* F_{Rij}\right] \\
&\times m_{\omega j} m_{\chi k} (A_1 - S_1) \\
&+ \mathrm{Im}\left[G_{Lki} G_{Rkj}^* F_{Rij} - G_{Rki} G_{Lkj}^* F_{Lij}\right] \\
&\times m_{\omega i} m_{\chi k} (-A_1 - S_1)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
f_{6(b)}^Z = &\frac{1}{(4\pi)^2} \frac{g^2}{\cos^2\theta_\mathrm{W}} \frac{1}{4} \sum_{i=1}^{4} \sum_{j=1}^{4} \sum_{k=1}^{2} \\
&\times \Big\{ \mathrm{Im}\left[G_{Lik} G_{Ljk}^* F_{ji} + G_{Rik} G_{Rjk}^* F_{ji}^*\right] \\
&\times \left(M_W^2 (A_3 - 2A_2) + 2q^2 A_4 + 3(m_{\chi i}^2 - m_{\chi j}^2) S_3\right) \\
&+ \mathrm{Im}\left[G_{Lik} G_{Ljk}^* F_{ji}^* + G_{Rik} G_{Rjk}^* F_{ji}\right] m_{\chi i} m_{\chi j} A_1 \\
&+ \mathrm{Im}\left[G_{Lik} G_{Rjk}^* F_{ji} + G_{Rik} G_{Ljk}^* F_{ji}^*\right] \\
&\times m_{\chi j} m_{\omega k} (-A_1 + S_1) \\
&+ \mathrm{Im}\left[G_{Lik} G_{Rjk}^* F_{ji}^* + G_{Rik} G_{Ljk}^* F_{ji}\right] \\
&\times m_{\chi i} m_{\omega k} (-A_1 - S_1)\Big\},
\end{aligned}
$$

Table 1. The values of $\tilde{m}_2$ and $\tan\beta$ for curves (i)–(iv) in Figs. 2 and 3

	(i)	(ii)	(iii)	(iv)
$\tilde{m}_2$ (GeV)	100	100	200	200
$\tan\beta$	2	10	2	10

$$
f_7^Z = 0, \tag{11}
$$

where S_i ($i = 1 - 3$) and A_i ($i = 1 - 4$) stand for the functions de ned by

$$
\begin{aligned}
[S_1, S_2, S_3] \equiv &\int_0^1 dx \int_0^1 dy \int_0^1 dz \ \frac{\delta(1 - x - y - z)}{D(m_i, m_j, m_k)} \\
&\times [1, z, xy],
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
[A_1, A_2, A_3, A_4] \equiv &\int_0^1 dx \int_0^1 dy \int_0^1 dz \ \frac{\delta(1 - x - y - z)}{D(m_i, m_j, m_k)} \\
&\times \left[x - y, z(x - y), z^2(x - y), xy(x - y)\right], \\
D(m_i, m_j, m_k) \equiv & -M_W^2\ z(1 - z) - q^2 xy + m_i^2 x + m_j^2 y \\
&+ m_k^2 z - i\varepsilon.
\end{aligned}
$$

For $f_{4(a)}^Z$ and $f_{6(a)}^Z$ the arguments of these functions are given by $m_i = m_{\omega i}$, $m_j = m_{\omega j}$, and $m_k = m_{\chi k}$, and for $f_{4(b)}^Z$ and $f_{6(b)}^Z$ given by $m_i = m_{\chi i}$, $m_j = m_{\chi j}$, and $m_k = m_{\omega k}$. The form factors $f_{i(a)}^Z$ and $f_{i(b)}^Z$ ($i = 4, 6$) arise from the diagrams in Fig. 1a and Fig. 1b, respectively.

ii) The $WW\gamma$ vertex:

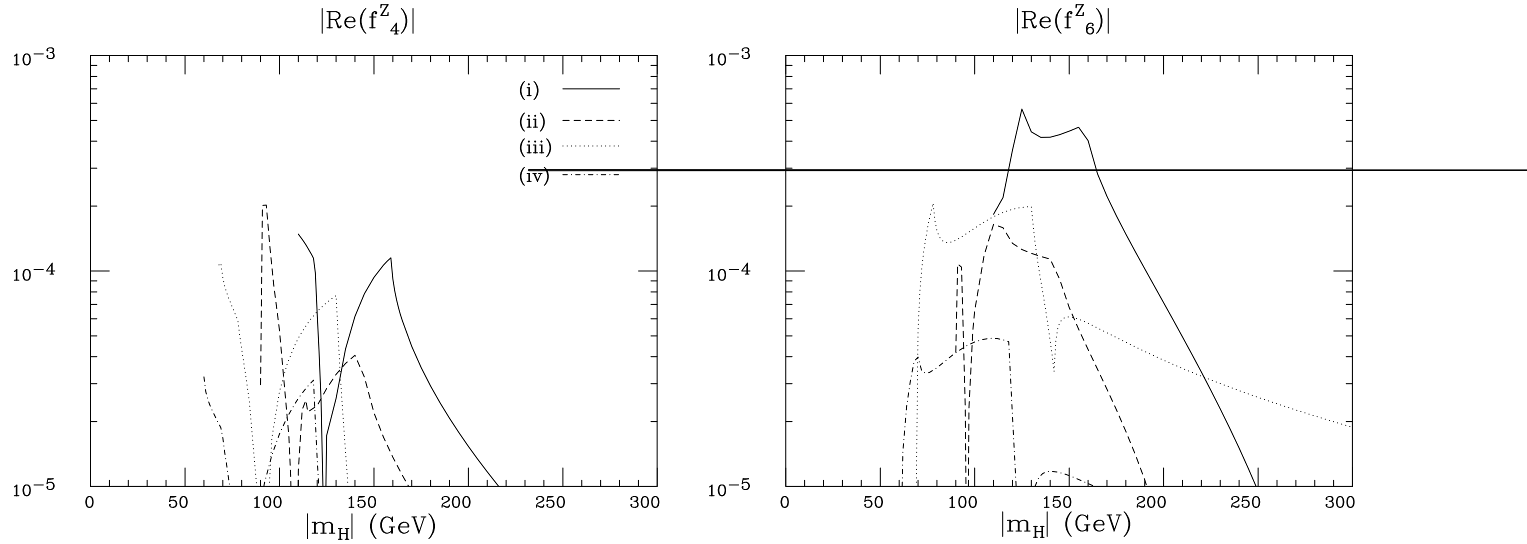
$$
\begin{aligned}
f_6^\gamma = &\frac{-1}{(4\pi)^2} g^2 \sum_{i=1}^{2} \sum_{k=1}^{4} \mathrm{Im}\Big[G_{Lki} G_{Rki}^* - G_{Lki}^* G_{Rki}\Big] \\
&\times m_{\omega i} m_{\chi k} S_1,
\end{aligned} \tag{13}
$$

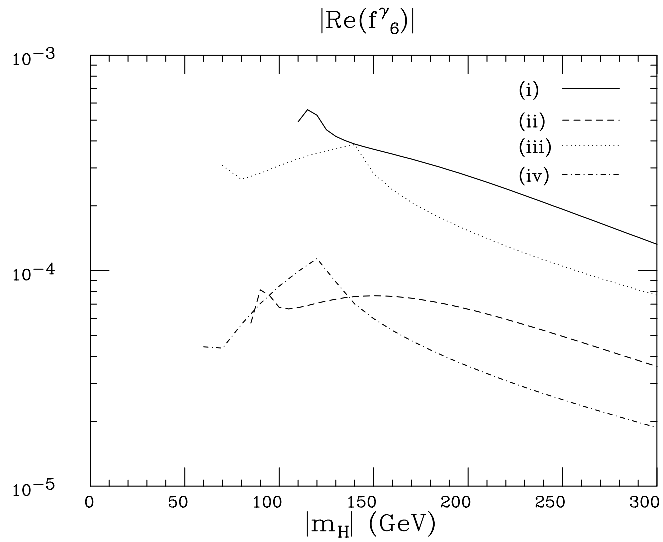
$$
f_4^\gamma = 0, \tag{14}
$$

$$
f_7^\gamma = 0. \tag{15}
$$

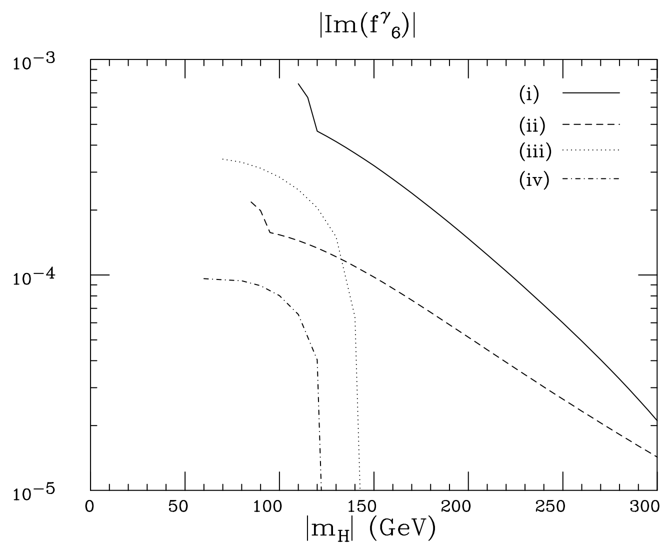
The arguments of S_1 are given by $m_i = m_j = m_{\omega i}$ and $m_k = m_{\chi k}$. The nonvanishing contribution to f_6^γ only comes from Fig. 1a with the charginos being the same $i = j$.

We now make numerical analyses of the form factors. The numerical computations of one-loop integrals have been carried out following the method of [14]. For the SM parameters we x $\sin^2\theta_W = 0.232$, $M_Z = M_W/\cos\theta_W = 91.2$ GeV, and $\alpha_{EM} = 1/128.9$. In Fig. 2 we show the absolute values of the real and imaginary parts of f_4^Z and f_6^Z as functions of the absolute value of the higgsino mass parameter m_H for four sets of values of $\tilde{m}_2$ and $\tan\beta$ listed in Table 1. For the CP-violating phase we take $\theta = \pi/4$. The value of the momentum-squared for the Z boson is set for $\sqrt{q^2} = 200$ GeV. In the ranges of smaller values for $|m_H|$ where curves are not drawn, the lighter chargino has a mass smaller than 45 GeV, which has been ruled out by LEP experiments [15]. If the masses of the particles which couple to the Z boson in a loop diagram are near the threshold, $\sqrt{q^2} \simeq m_i + m_j$, the contributions of this diagram to the form factors are enhanced. Below


|Re(f^Z_4)|
(i)
(ii)
(iii)
(iv)
10^-3
10^-4
10^-5
0
50
100
150
200
250
300
|m_H| (GeV)
|Re(f^Z_6)|
10^-3
10^-4
10^-5
0
50
100
150
200
250
300
|m_H| (GeV)



a



b

Fig. 3. The absolute values of the real and imaginary parts of f_6^γ as functions of $|m_H|$ for $\theta = \pi/4$ at $\sqrt{q^2} = 200$ GeV. Four curves (i)–(iv) correspond to the four sets of parameter values given in Table 1. **a** $\mathrm{Re}(f_6^\gamma)$, **b** $\mathrm{Im}(f_6^\gamma)$

Table 2. The mass spectra of the charginos and neutralinos in Figs. 4 and 5

$m_{\omega i}$ (GeV)	133	274		
$m_{\chi j}$ (GeV)	85	146	203	278

catedly, while f_6^γ does simply. The magnitudes of $\mathrm{Re}(f_6^Z)$, $\mathrm{Im}(f_6^Z)$, $\mathrm{Re}(f_6^\gamma)$, and $\mathrm{Im}(f_6^\gamma)$ can become around 2×10^{-4}, though those of $\mathrm{Re}(f_4^Z)$ and $\mathrm{Im}(f_4^Z)$ are at most 2×10^{-5}. If $\tilde{m}_2$ and $|m_H|$ are of order 100 GeV and $\tan\beta$ is not much larger than unity, in general, there is a region of $\sqrt{q^2}$ where $|f_6^Z|$ or $|f_6^\gamma|$ becomes larger than 1×10^{-4}.

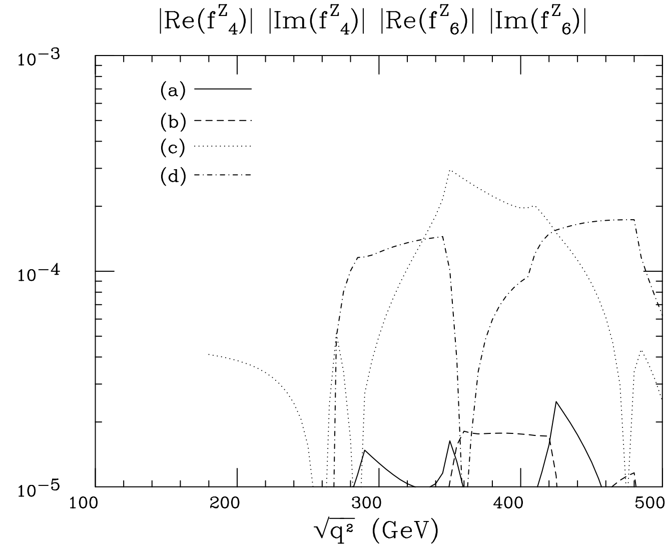


Fig. 4. The absolute values of the real and imaginary parts of f_4^Z and f_6^Z as functions of $\sqrt{q^2}$ for $\tan\beta = 2$, $\tilde{m}_2 = 200$ GeV, $|m_H| = 200$ GeV, and $\theta = \pi/4$. (a) $\mathrm{Re}(f_4^Z)$, (b) $\mathrm{Im}(f_4^Z)$, (c) $\mathrm{Re}(f_6^Z)$, (d) $\mathrm{Im}(f_6^Z)$

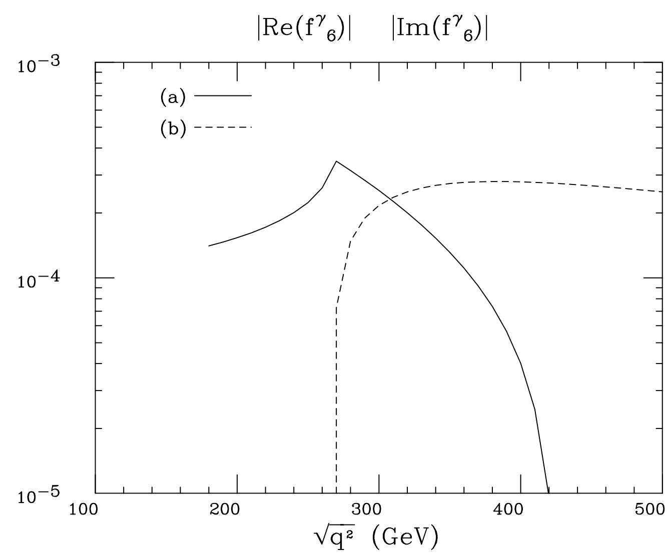


Fig. 5. The absolute values of the real and imaginary parts of f_6^γ as functions of $\sqrt{q^2}$ for $\tan\beta = 2$, $\tilde{m}_2 = 200$ GeV, $|m_H| = 200$ GeV, and $\theta = \pi/4$. (a) $\mathrm{Re}(f_6^\gamma)$, (b) $\mathrm{Im}(f_6^\gamma)$

4 Discussions

We have shown that the SSM yields CP-odd anomalous couplings for the WWZ and $WW\gamma$ vertices at the one-loop level. The CP-odd form factors could have magnitudes of $10^{-3} - 10^{-4}$, which are far larger than the SM predictions. If some CP-violating phenomena originating from the CP-odd couplings are observed, the SSM would become more promising as physics beyond the SM.

We now discuss the e ects of the CP-odd form factors on observable quantities. The resultant CP-violating phenomena occur primarily in the pair production of polarized W bosons in e^+e^- annihilation, $e^+e^- \to W^+(\lambda)$ $W^-(\bar{\lambda})$, λ and $\bar{\lambda}$ denoting respectively the helicities of W^+ and W^-. Among various combinations for the helicities $(\lambda, \bar{\lambda})$, the pairs $(+, 0)$, $(-, 0)$, and $(+, +)$ are CP-conjugate to the pairs $(0, -)$, $(0, +)$, and $(-, -)$, respectively. If CP invariance is conserved, these CP-conjugate processes have the same cross sections. Thus, nonvanishing values for the di erences of the cross sections $\sigma_{+0} - \sigma_{0-}$, $\sigma_{-0} - \sigma_{0+}$, and $\sigma_{++} - \sigma_{--}$ exhibit CP violation. These di erences are indeed generated by the imaginary parts of the CP-odd form factors. For instance, CP violation can be evaluated by an asymmetry

$$A_{CP} = \frac{\sigma_{+0} - \sigma_{0-}}{\sigma_{+0} + \sigma_{0-}}, \tag{16}$$

which is given by

$$\begin{aligned} A_{CP} &= \left(-1 + \frac{s}{s - M_Z^2}\right)^{-1} \left\{ 2I^\gamma + \frac{s}{s - M_Z^2}(I^\gamma + I^Z) \right. \\ &\quad \left. + \left(\frac{s}{s - M_Z^2}\right)^2 2I^Z \right\}, \\ I^V &= \mathrm{Im}(f_4^V) - \frac{\mathrm{Im}(f_6^V)}{\beta}, \end{aligned} \tag{17}$$

where $\beta = \sqrt{1 - 4M_W^2/q^2}$. The calculation has been performed for polarized electron and positron beams. We can see that the magnitude of A_{CP} becomes of order of the CP-odd form factors. In the SSM such CP asymmetries could thus be of order of $10^{-3} - 10^{-4}$ in a maximal case. On the other hand, the real parts of the CP-odd form factors induce T violation in the angular distribution of the polarization vector $\boldsymbol{\epsilon}$ for the W^+ or W^- boson, leading to a T-odd asymmetry [8,16]

$$A_T = \frac{\sigma((\mathbf{p}_- \times \mathbf{p}) \cdot \boldsymbol{\epsilon} > 0) - \sigma((\mathbf{p}_- \times \mathbf{p}) \cdot \boldsymbol{\epsilon} < 0)}{\sigma((\mathbf{p}_- \times \mathbf{p}) \cdot \boldsymbol{\epsilon} > 0) + \sigma((\mathbf{p}_- \times \mathbf{p}) \cdot \boldsymbol{\epsilon} < 0)}, \tag{18}$$

where $\mathbf{p}_-$ and $\mathbf{p}$ denote the momenta of the electron and the W boson, respectively. The value of A_T becomes also the same order of magnitude as the CP-odd form factors.

The helicity of the W boson a ects the energy distribution of the particle produced by the W-boson decay. Consequently the CP asymmetry A_{CP} for the W-boson pair production could be observed as an asymmetry between the energy distributions of the particles produced from W^+ and W^-. Unless the contributions of di erent CP-conjugate pairs are canceled, this resultant asymmetry would be of the same order of magnitude as A_{CP}. The T-odd asymmetry A_T leads to some T-odd asymmetry among the particle momenta in the nal state [11] with the same order of magnitude. Assuming maximal CP violation, a total of $10^6 - 10^8$ pairs of W bosons would make it possible to examine these asymmetries. However, in near-future e^+e^- experiments it seems to be di cult to achieve such a number of events [17]. A nonvanishing value of f_6^γ leads to the EDMs of the neutron and the electron at the two-loop level [10,18], which might be accesible by possible improvement for precision of EDM experiments [19].

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